

**WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH
 PROBLEM SET V (2016-2017) February 2017**

1. Find all solutions of the following equation:

$$|\dots||x| - 1| - 2| - \dots| - 2017| = 0.$$

(On the left side of the equation we start with $|x|$, subtract 1, take absolute values, subtract 2, take absolute values, subtract 3, and so on.)

2. Consider a circle, and three points, A, B, C on the circle. Denote by P the intersection point of the tangent line to the circle at the point C with the line AB. Let Q be the intersection of line AB and the angle bisector of angle $\angle ACB$. Show that $PQ = PC$.
3. We have a 2×13 table filled with integers. Each cell contains exactly one number. The numbers in the first row are red and the numbers in the second row are blue. Show that we can choose 3 columns so that both the red numbers and the blue numbers in the chosen columns have integer valued averages.
4. We have a sequence of integers a_1, a_2, \dots such that $a_1 = 2$ and $a_{n+1} = a_n^2 - a_n + 1$ for all $n \geq 1$. What is the 1000th digit after the decimal place in the decimal representation of

$$\frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_{2017}}?$$

5. Cheryl chose three distinct integers between 1 and 5. We would like to identify these numbers. We are allowed to make queries in the following form: we give Cheryl 3 distinct numbers a, b , and c , and she tells us how many of these numbers are among chosen ones. For example, if we guess 2, 3, and 4 and the chosen numbers are 1, 2, and 5, we get the answer 1. Find the smallest number of queries needed to guarantee that we can always identify the three chosen numbers. Note that we do not need to actually name the three correct numbers in a query; it is sufficient to identify them without a doubt from the replies to our queries.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

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