

WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH

PROBLEM SET IV (2017-2018)

January 2018

1. Find the number of pairs of positive integers m and n such that $m + n = 2018$, and the addition of m and n involves no carry. (For example, in the case of $1012 + 1006$ there is no carry in the addition, but in the case of $1009 + 1009$ there is a carry from the ones place to the tens place.)
2. For which values of $n \geq 1$ is the expression $2018^n + 2019^n + 2020^n + 2021^n + 2022^n$ divisible by 5?
3. Show that if n is a positive integer, then

$$\sqrt[n]{n+1} < 1 + \frac{\sqrt{2}}{\sqrt{n}}.$$

4. Use each of the digits 4, 5, 6, 7, 8, 9 exactly once to form two three digit numbers having the largest possible product. (Don't forget to give a complete justification that you have found the largest possible product.)
5. Hexagon $ABCDEF$ is inscribed in a circle radius R . Its side lengths are $AB = CD = EF = 4$ and $BC = DE = FA = 13$. Find R . For full credit, your proof should not include any trigonometry.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

Find old and current problems and information about the talent search at: <http://www.math.wisc.edu/talent>

Find an introduction to techniques for solving problems like these at: <https://goo.gl/pqq32m>

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