1. We are given a 2020-sided convex polygon. We want to select three distinct edges of the polygon, so that if we go around the edges in clockwise order, at least two unselected edges lie between every pair of selected edges. In how many different ways can we select the three edges?

2. On quadrilateral $ABCD$, points $E$, $F$, $G$, and $H$ are the midpoints of sides $AB$, $BC$, $CD$, and $DA$, respectively. Suppose the diagonals $AC$ and $BD$ of quadrilateral $ABCD$ intersect at the same point as the diagonals $EG$ and $FH$ of quadrilateral $EFGH$. Show that $ABCD$ must be a parallelogram.

3. In triangle $\triangle PQR$, the midpoint of side $QR$ is denoted by $S$. Find $\angle QPR$, if we know that $\angle PRQ = 30^\circ$ and $\angle PSQ = 45^\circ$.

4. We construct a sequence of prime numbers $p_1, p_2, \ldots$, as follows: We set $p_1 = 2$. For any $n \geq 1$, the integer $p_{n+1}$ is the largest prime factor of the number which is one larger than the product of $p_1, \ldots, p_n$. (So for example, $p_4$ is the largest prime factor of $1 + p_1 \cdot p_2 \cdot p_3$.) Show that $p_n \neq 5$ for all $n \geq 1$.

5. The edge lengths of a triangle are given by $a, b,$ and $c$. We know that $ab + bc + ac = 12$. Show that the perimeter of the triangle cannot be larger than 7.