

**WISCONSIN MATHEMATICS, SCIENCE & ENGINEERING TALENT SEARCH  
 PROBLEM SET V (2018-2019) February 2019**

1. Suppose we have 100 special points in space, such that no 3 special points lie on the same line. Show that we can always draw 2019 line segments, each of which has two special points as endpoints, in such a way that the segments do not form any triangle with 3 special points as vertices.
2. We call a positive integer balanced if its decimal digits can be divided into two groups so that the sums of digits in the two groups are equal (e.g., 22, 101, and 134 are all balanced). Find the smallest positive integer  $n$  such that both  $n$  and  $n + 1$  are balanced.
3. Inside the regular hexagon  $ABCDEF$ , we use red to color the points that are closer to the diagonal  $\overline{AD}$  than to the outside of the hexagon, and use blue to color the remaining region. Find the ratio of the area of the red region to the area of the blue region.
4. We have 2019 distinct points  $A_1, A_2, \dots, A_{2019}$  in the plane, such that the points do not all lie on the same line. Two different points  $P$  and  $Q$  exist that satisfy

$$A_1P + \dots + A_{2019}P = A_1Q + \dots + A_{2019}Q.$$

Show that a point  $R$  must exist that satisfies

$$A_1R + \dots + A_{2019}R < A_1P + \dots + A_{2019}P.$$

5. We write down the numbers  $(44 + \sqrt{2019})^n$ , for  $n = 1, 2, 3, \dots$ , in order, and then delete all decimal digits after the decimal points. Show that in the resulting sequence of integers, the even and odd numbers alternate.

You are invited to submit a solution even if you get just one problem. Please do not write your solutions on this problem page. Remember that solutions require a proof or justification.

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Find an introduction to techniques for solving problems like these at: <https://goo.gl/pqq32m>

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