1. Show that the following two definitions of quasi-separated-ness of a scheme $S$ are equivalent:
   (1) The intersection of any two quasi-compact open subsets of $S$ is quasi-compact;
   (2) There is a cover of $S$ by affine open subsets whose (pairwise) intersections are quasi-compact.

2. In class, we gave the following definition: a scheme $S$ is integral if it is irreducible and reduced. Show that this is equivalent to the definition from Vakil’s notes: a scheme is integral if for any non-empty open $U \subset S$, $O_S(U)$ is a domain.

3. Let us call a scheme $X$ locally irreducible if every point has an irreducible neighborhood. (Since a non-empty open subset of an irreducible space is irreducible, this implies that all smaller neighborhoods of this point are irreducible as well.) Prove or disprove the following claim: a scheme is irreducible if and only if it is connected and locally irreducible.

4. Show that a locally Noetherian scheme is quasi-separated.

5. Show that the following two definitions of a Noetherian scheme $X$ are equivalent:
   (1) $X$ is a finite union of open affine sets, each of which is the spectrum of a Noetherian ring;
   (2) $X$ is quasi-compact and locally Noetherian.

6. Show that any Noetherian scheme $X$ is a disjoint union of finitely many connected open subsets (the connected components of $X$.) (A problem from the last homework shows that things might go wrong if we do not assume that $X$ is Noetherian.)

7. A locally closed subscheme $X \subset Y$ is defined as a closed subscheme of an open subscheme of $Y$. Accordingly, a locally closed embedding is a composition of a closed embedding followed by an open embedding (in this order). In principle, one can try to reverse the order, and consider open subschemes of closed subschemes of $Y$. Does this yield an equivalent definition?

   Remark. The difficulty of such questions (and, sometimes, the answer to them) depends on the class of schemes one works with: often, very mild assumptions (such as, say, quasicompactness) would make the question easy. A complete answer to this problem would include both the mild assumptions that would make the two versions equivalent, and a description of what happens for general schemes.