

INTRODUCTORY PROBLEMS (09/11/13)

WARM-UP

(From *Problem-Solving Strategies* by A. Engel)

1. A rectangular floor is covered by 2×2 and 1×4 tiles. One tile got smashed, but there is a tile of the other kind available. Show that the floor cannot be covered by rearranging the tiles.
2. In how many ways can you select two disjoint subsets from an n -set?

SOME VT PROBLEMS

(Chosen quasi-randomly from past exams on the official site.)

3. A person is engaged in working a jigsaw puzzle that contains 1000 pieces. It is found that it takes 3 minutes to put the first two pieces together and that when x pieces have been connected it takes $3(1000-x)/(1000+x)$ minutes to connect the next piece. Determine an accurate estimate of the time it takes to complete the puzzle. Give both a formula and an approximate numerical value in hours. (You may find useful the approximate value $\ln(2) = .69$)
4. Solve the initial value problem

$$\frac{dy}{dx} = y \ln(y) + ye^x, \quad y(0) = 1$$

A FEW (EASIER) PUTNAM PROBLEMS

(Found at <https://www.math.okstate.edu/~wrightd/putnam/putnamprobs/>).

5. How many positive numbers n divide at least one of the numbers 10^{40} , 20^{30} ?
6. Let n be a positive integer, and define

$$f(n) = 1! + 2! + \cdots + n!.$$

Find polynomials $P(x)$, $Q(x)$ such that

$$f(n+2) = P(n)f(n+1) + Q(n)f(n)$$

for all $n \geq 1$.

7. Let A be a solid $a \times b \times c$ rectangular brick in three dimensions, $a, b, c > 0$. Let B be the set of all points that are at a distance at most 1 from some point of A ; in particular, B contains A . Express the volume of B as a polynomial in a , b , and c .

UW Putnam Club

Meeting time: Wednesday 5–6:30pm, VV B 115.

Putnam competition: First Saturday in December (December 6, 2014). Two three-hour sessions of six problems each. Over 2,000 college students participate; there is also an official UW team (3 students).

Virginia Tech Regional Math Competition: 9–11:30 am, October 25, 2014, 7 problems. More than 600 contestants from over 100 schools. Kind of ‘Putnam preparation’, somewhat easier.

Common topics: Linear algebra, elementary number theory, calculus, combinatorics; emphasis on problem-solving.