1. Each of the faces of the cube is colored a different color. How many of the colorings are distinct?

2. Find the sum
\[ \sum_{k=1}^{n} \binom{n}{k} k^3. \]
(Consider the problem of selecting a committee, and a chairman, vice-chairman, and a secretary in this committee.)

3. How many ways are there to choose \( n \) objects from \( 3n + 1 \) objects, assuming that of these \( 3n + 1 \), \( n \) objects are indistinguishable, and the rest are all distinct?

4. How many subsets of \( \{1, \ldots, n\} \) have no two successive numbers?

5. Can we arrange the numbers 1, 2, \ldots, 9 along a circle so that the sum of two neighbors is never divisible by 3, 5, or 7?

6. Consider a circular row of \( n \) seats; a child seats on each. Each child can move by at most one seat. Find the number of ways in which they can rearrange.

7. Is there a subset \( A \subset \{1, \ldots, 3000\} \) with 2000 elements such that if \( x \in A \), then \( 2x \notin A \).

8. Does a polyhedron exist with an odd number of faces, each face having an odd number of edges?

9. Let \( 1 \leq r \leq n \) and consider all subsets of \( r \) elements of the set \( \{1, 2, \ldots, n\} \). Each of these subsets has a minimal element. Let \( F(n, r) \) denote the mean of these smallest numbers; show that
\[ F(n, r) = \frac{n + 1}{r + 1}. \]