1. Find all functions $f : \mathbb{R} \to \mathbb{R}$ satisfying
   \[ f(x^2 - y^2) = (x - y)(f(x) + f(y)). \]

2. Find all functions $f : \mathbb{R} - \{1\} \to \mathbb{R}$, continuous at 0, that satisfy
   \[ f(x) = f\left(\frac{x}{1 - x}\right), \quad x \in \mathbb{R} - \{1\}. \]

3. Find all functions $f : [0, 1] \to \mathbb{R}$ satisfying the following conditions
   \begin{itemize}
   \item $[f(x)] \sin^2 x = [x] \cos f(x) \cos x = f(x)$
   \item $f(f(x)) = f(x)$.
   \end{itemize}

   Here $[x]$ means the fractional part of $x$.

4. Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that for all reals $x, y, z$, we have
   \[ [f(x) + 1][f(y) + f(z)] = f(xy + z) + f(xz - y). \]

5. Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that
   \[ f(f(x) + y) = f(f(x) - y) + 4f(x)y, \quad \text{for any } x, y \in \mathbb{R}. \]

6. Let $c$ be a positive integer. The sequence $a_1, a_2, \ldots$ is defined by
   \[ a_1 := c, \quad \text{and } a_{n+1} = a_n^2 + a_n + c^3, n \in \mathbb{N}. \]

   Find all values of $c$ for which there exist some integers $k \geq 1$ and $m \geq 2$ such that $a_k^m = c^3$ is the $m$-th power of some positive integer.

7. (Putnam) Find all functions $f$ from the interval $(1, \infty)$ to $(1, \infty)$ with the following property: if $x, y \in (1, \infty)$ and $x^2 \leq y \leq x^3$, then $(f(x))^2 \leq f(y) \leq (f(x))^3$.

8. Find all differentiable functions $f : (0, \infty) \to (0, \infty)$ for which there is a positive real number $a$ such that
   \[ f'(\frac{a}{x}) = \frac{x}{f(x)}, \]
   for all $x > 0$. 