1. Determine the number of possible values for the determinant of $A$, given that $A$ is a $n \times n$ matrix with real entries such that $A^3 - A^2 - 3A + 2I = 0$, where $I$ is the identity and $0$ is the all-zero matrix.

2. Consider matrices $A_1, A_2, \ldots, A_m \in M_n(\mathbb{R})$ not all nilpotent. Prove that there is an integer number $k > 0$ such that $A_1^k + A_2^k + \ldots + A_m^k \neq O_n$.

3. Let $A, B \in M_2(\mathbb{C})$ such that $A^2 + B^2 = 2AB$.
   a) Prove that $AB = BA$.
   b) Prove that $\text{tr}(A) = \text{tr}(B)$.

4. Let $A, B$ be matrices of dimension $2010 \times 2010$ which commute and have real entries, such that $A^{2010} = B^{2010} = I$, where $I$ is the identity matrix. Prove that if $\text{tr}(AB) = 2010$, then $\text{tr}(A) = \text{tr}(B)$.

5. Let $A$ and $B$ be real symmetric matrices with all eigenvalues strictly greater than $1$. Let $\lambda$ be a real eigenvalue of matrix $AB$. Prove that $|\lambda| > 1$.

6. Does there exist a real $3 \times 3$ matrix $A$ such that $\text{tr}(A) = 0$ and $A^2 + A^t = I$? (tr($A$) denotes the trace of $A$, $A^t$ the transpose of $A$, and $I$ is the identity matrix.)

7. Let $A, B \in M_n(\mathbb{C})$. If $A(AB - BA) = (AB - BA)A$ prove that $AB - BA$ is nilpotent.

8. Let $A$ be a real symmetric matrix and $B \in M_n(\mathbb{C})$ such that $AB + BA = 0$. Prove that $AB = 0$.

9. Let $A \in \text{Sl}_3(\mathbb{Z})$ of finite order. Find all possible values of $\text{tr}(A)$.

10. Let $A$ and $B$ be two complex matrices. Prove that the following conditions are equivalent:
    a) For any $M \in M_n(\mathbb{C})$ the characteristic polynomials of $AM$ and $AM + B$ are the same
    b) $B$ is nilpotent and $BA = O_n$.

11. $A, B \in M_2(\mathbb{C})$ with $\det(A) = 1, -\text{tr}(A) \neq 2, \det(B) = 1, |\text{tr}(B)| \neq 2$ and suppose also $A, B$ do not have common eigenvectors. Given that there exist $(n_1, \ldots, n_k, m_1, \ldots, m_k) \in \mathbb{Z}$ such that $A^{n_1}B^{m_1} \ldots A^{n_k}B^{m_k} = I_2$ prove that $A^{-n_1}B^{-m_1} \ldots A^{-n_k}B^{-m_k} = I_2$.

12. Let $A, B \in M_2(\mathbb{C})$ with $\exp(A) = \exp(B)$. Suppose for any eigenvalue $a$ of $A$ and $b$ of $B$, $a - b \neq 2\pi i \mathbb{Z}$. Then $A = B$.

13. Let $A, B$ be square matrices a) of size $2016 \times 2016$; b) of size $2017 \times 2017$. Do there necessarily exist real numbers $a, b$ such that $a^2 + b^2 \neq 0$ and the matrix $aA + bB$ is singular?

14. Let a square matrix $P$ be neither zero nor unit and such that $P^2 = P$. Does there always exist such a matrix $Q$ that $Q^2 = Q$, $PQ = QPQ$ but $QP \neq PQ$?