

Introduction

- 1.** Prove that every (positive) composite integer n can be written as $n = xy + xz + yz + 1$ for some positive integers x , y , and z . (Putnam 1988)
- 2.** The number of distinct positive divisors of a positive integer n is a prime. Show that n is an integer power of a prime.
- 3.** Given any five points on a sphere, show that some four of them must lie on a closed hemisphere. (Putnam 2002)
- 4.** Show that $1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$ is never an integer for $n > 1$.
- 5.** Let $p(x)$ be a polynomial with integer coefficients such that $p(0)$ and $p(1)$ are both odd. Show that $p(x)$ has no integer roots.
- 6.** Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be continuous, and suppose that there is some real number a such that $f(f(f(a))) = a$. Show that there is some real number b such that $f(b) = b$.