1. Determine the number of ordered triples \((A_1, A_2, A_3)\) of sets such that \(A_1 \cup A_2 \cup A_3 = \{1, 2, \ldots, 10\}\) and \(A_1 \cap A_2 \cap A_3 = \emptyset\). (Putnam 1985)

2. Show that \(\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}\).

3. (a) Show that \(\sum_{k=1}^{n} k \binom{n}{k} = n2^{n-1}\).

(b) Show that \(\sum_{k=1}^{n-1} \sum_{j=1}^{n-k} kj \binom{n}{k} \binom{n-k}{j} = n(n-1)3^{n-2}\).

4. How many ways are there to write \(n\) as the ordered sum of positive integers? (E.g., if \(n = 4\), the different sums are 4, 3+1, 1+3, 2+2, 1+1+2, 1+2+1, 2+1+1, and 1+1+1+1.)

5. 14 positive integers are chosen less than 1000. Show that there is some pair of disjoint subsets of these numbers whose sums are equal. (Hint: use counting and pigeonhole.)

6. The numbers 1, 2, \ldots, 3n + 1 are written in a random order. What is the probability that the sum of the first \(k\) of them (for each \(k = 1, 2, \ldots, 3n + 1\)) is never divisible by 3? (Putnam 2007)