

NAME:

STUDENT ID:

INSTRUCTOR:

TA:

Grading Table

Question	Possible Points	Points Earned
1	15	
2	10	
3	15	
4	15	
5	15	
6	15	
7	15	
EC	10	
Total	100	

Please read these instructions carefully before beginning.

1. Do not open the exam until 5:30. You will have 90 minutes to complete the exam.
2. Final answers must be written clearly in the proper space provided. No credit will be given for illegible or ambiguous answers. Answers with no justification will not be given credit.
3. No notes, calculators, or electronic devices are permitted.

1. (15 points) Compute the following limits:

(a)

$$\lim_{x \rightarrow -4} \frac{x^2 + 2x - 8}{x + 4}$$

(b)

$$\lim_{x \rightarrow -4^-} \frac{x^2 + 2x + 8}{x + 4}$$

(c)

$$\lim_{x \rightarrow -4^+} \frac{x^2 + 2x + 8}{x + 4}$$

(d)

$$\lim_{x \rightarrow -4} \frac{x^2 + 2x + 8}{(x + 4)^4}$$

2. (10 points) For each of parts (a) and (b), write a formula for a function $f(x)$ with the characteristics specified:

(a)

$$\lim_{x \rightarrow \pm\infty} f(x) = 3$$

$$\lim_{x \rightarrow -2^+} f(x) = -\infty$$

$$\lim_{x \rightarrow -2^-} f(x) = \infty.$$

(b)

$$\lim_{x \rightarrow \pm\infty} f(x) = 3$$

$$\lim_{x \rightarrow 4} f(x) = -\infty$$

3. (15 points) Let $f(x) = \sqrt{2x - 3}$.

Using the definition of derivative, find $f'(x)$.

4. (15 points) Use implicit differentiation to find the following derivatives

(a) Find $\frac{dy}{dx}$ of

$$5y^2 - 2x = x(x^2y + 3y)$$

(b) Find $\frac{dr}{dt}$ of

$$3r^2t - r^2 = t^3r + t^2r$$

5. (15 points) Compute the following derivatives:

(a)

$$\frac{d}{dx} \left(\frac{(x + x^3)^7}{x + (x^3 - 4)^2} \right)$$

(b)

$$\frac{d}{ds} ((5x + 3)^8(7x - 2)^5)$$

6. (15 points) Prove by definition of limit, that

$$\lim_{x \rightarrow 3} (7x - 2) = 19$$

7. (15 points) Suppose that you have a cube with edge length l . Write an equation describing the relationship between l and the volume of the cube. If the side length is expanding at a rate of 10 cm/sec, how fast is the volume changing when the edge length is 100 cm.

Extra Credit (10 points) Suppose that $u(x)$ and $v(x)$ are two differentiable functions. Using the values for u, v, u', v' , below, compute the following derivatives:

(a) $\frac{d}{dx}u \circ v$ at $x = 2$

(b) $\frac{d}{dx}v \circ u$ at $x = 1$

(c) $\frac{d}{dx}uv$ at $x = 1$

(d) $\frac{d}{dx}\frac{u}{v}$ at $x = 2$

(e) $\frac{d}{dx}\frac{v}{u}$ at $x = 1$

Here is a table containing the relevant values for u and v .

$$\begin{array}{l} u(1) = 3 \quad u'(1) = 2 \quad v(1) = 3 \quad v'(1) = 8 \\ u(2) = 5 \quad u'(2) = 9 \quad v(2) = 1 \quad v'(2) = 4 \\ u(3) = 6 \quad u'(3) = 5 \quad v(3) = 2 \quad v'(3) = 7 \end{array}$$