Math 571: Mathematical Logic
0th homework set, due at 2:25pm on Monday, Sept. 9.
Bring your solutions to class, or slide them under the door of Van Vleck 513.

1. (Here, I make sure that you know what a proof is.)
   Write a careful, complete, coherent proof of each of the following statements. Write complete sentences; include all details; start by explaining what the statement means.

   (a) There are infinitely many primes.

   (b) The sum of the cubes of the first n positive integers is the square of the sum of these numbers.

   (c) (Harder) Write out a careful proof of the Schöder-Bernstein theorem.

2. (Here, you begin thinking about formal logic. These will be entirely automatic very soon.)
   Decide whether the two statements in each pair mean the same thing. If you think they do, explain why. If you think they don't, describe a situation where one statement is true and the other is not.

   (a)   i. If it rains next sunday, then Andy will only go to the football game if Bob does.

         ii. If Andy will only go to the foot ball game on Sunday if it rains, then Bob will definetely go to the Sunday football game regardless of the weather.

   (b)   i. All meeps are moops, nothing yellow is a meep, nothing yellow is a moop unless it is also a meep, and some moops aren't meeps.

         ii. No moop is yellow, not every moop is a meep, but every meep is a moop.

3. (Here is a cute riddle about infinite processes.)
   A train with infinitely many seats, one for each rational number, stops in countably many villages, one for each positive integer, in increasing order, and then finally arrives at the city. (For example, the distances between villages could be getting smaller, or the speed of the train faster, so that the train stops at the first village at noon, at the second village at 1pm, and the third at 1:30pm, and the fourth at 1:45pm, and so on until it arrives at the city at 2:17pm.) At the first village, two women board the train. At the second village, one woman leaves the train to go visit her cousin, and two other women board the train. At the third village, one woman leaves the train to go visit her cousin, and two other women board the train. At the fourth village, and in fact at every later village, the same thing keeps happening: one woman off to visit her cousin, two new women on board the train. How many women arrive at the city? Explain your answer!