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Answer all four questions below. Show your working. Full credit will not be given for just the answer without any justification. Make sure you answer each part of each question.

1. (a) How many keys are there for (i) shift ciphers, (ii) affine ciphers, (iii) general substitution ciphers, (iv) Vigenere ciphers where the keyword has length 3?
   (b) An affine cipher has encryption function $e_k(x) = 3x + 1 \pmod{26}$. What is its decryption function?

2. (a) The ciphertext DRZNUO was produced by using a Vigenere cipher with keyword BABY. What is the blocklength of the cipher? What is the plaintext?
   (b) Encrypt BABY using the Hill cipher with key $\begin{pmatrix} 3 & 2 \\ 13 & 9 \end{pmatrix}$. What is the decrypting key?

3. What are the three kinds of attack you have met so far? Define each one. Suppose we have an affine cipher and we know how A and B are encrypted - show that this is enough to find the key. Suppose that instead we only know how A and C are encrypted - show that this is not enough to find the key uniquely [hint: say A is encrypted as B and C encrypted as D - what two possible keys are there?]

4. Describe briefly how frequency analysis is used to break a monoalphabetic cipher. In particular, which of the three kinds of attack is this used for? The most commonly used letters in English are E and then T. The ciphertext PDAPAOP was produced using a shift cipher. Using frequency analysis find the plaintext (which should make sense in English).