Please solve the following problems taken from Barg’s notes.

1. Page 20: Show that the sum of two even-weighted vectors has even weight.

2. Page 25: Show that Hamming distance is a translation-invariant metric (if you don’t know the definition of metric, just email me - translation-invariant means that \( d(u + e, v + e) = d(u, v) \) for all vectors \( u, v, e \)).

3. Page 35: Suppose that the rank of \( H(E) \) is less than \(|E|\). Is there a codeword \( x \) with \( \text{supp}(x) = E \)? (Hint: first, look for counterexamples.)

4. Page 49: Show that the set of correctable errors \( E(C) \) are the unique vectors of smallest weight in their cosets.


6. Page 64: Is the all one vector (11...1) a codeword of the \( m \)th generalized Hamming code for every \( m \)?