1. Recovering a function from its Radon transform is an important technique in applied statistics. The question of invertibility of a particular kind of discrete Radon transform amounts to finding solutions of the Diophantine equation

\[ 3(y^2 - 1) = 2x^2(x^2 - 1) \quad (*) \]

(a) Find a birational transformation that turns this equation into Weierstrass form.
(b) Use the tangent-chord method to find at least 17 solutions in rational numbers.
(c) Look up Mazur’s theorem on the torsion subgroup of an elliptic curve over \( \mathbb{Q} \) and use this to say how many solutions there are to \((*)\) with \( x, y \in \mathbb{Q} \).
(d) It has been shown that all solutions in integers to \((*)\) have \( |x| \) in a set \( S \) of size 6. Find \( S \). What is interesting about \( \{(2x - 1)^2 : x \in S\} \)?