1. (i) Let $E$ be an elliptic curve over a field $K$ of characteristic $p > 0$. Assuming $E[p]$ has order 1 or $p$ (proven later in the course), bound $[K(E[p]) : K]$. Is $K(E[p]) / K$ always separable?

(ii) Let $K$ be a field of characteristic two. Let $E$ be the curve $y^2 + xy = x^3 + ax^2 + b$, where $a, b \in K, b \neq 0$. Show that $E$ is an elliptic curve. If $P$ is the point $(u, v)$ on $E$, find a formula for $2P$.

(iii) For $K$ and $E$ as in (ii), find $E[2], K(E[2]), E[4]$, and $K(E[4])$. What is $[K(E[4]) : K]$ and is it always a separable extension?