Let $E$ be the elliptic curve $y^2 + y = x^3 - x^2$ defined over $\mathbb{Q}$. Let $\rho_p : G_\mathbb{Q} \to GL_2(\mathbb{F}_p)$ denote the associated Galois action on $E[p]$.

(a) Find an equation for the $x$-coordinates of the points in $E[2]$. Find the image of $\rho_2$.

(b) Find an equation for the $x$-coordinates of the points in $E[3]$. Show that the only subgroup of $GL_2(\mathbb{F}_3)$ that surjects onto $PGL_2(\mathbb{F}_3)$ is $GL_2(\mathbb{F}_3)$. Find the image of $\rho_3$. Does $E$ have complex multiplication?

(c) Find a point in $E(\mathbb{Q})$ of order 5. What does this tell us about the image of $\rho_5$?