1. (i) For the elliptic curve \( E : y^2 = 4x^3 - g_2x - g_3 \), let \( g_2, g_3 \to 0 \). Show that the same geometric procedure for finding \( P_1 + P_2 \) on \( E \) makes the smooth points of the curve \( y^2 = 4x^3 \) into an abelian group isomorphic to the additive group of \( \mathbb{C} \). Interpret this in terms of what happens to the lattice and a fundamental parallelogram.

(ii) For the same elliptic curve \( E \), let \( g_2 \to 4/3 \) and \( g_3 \to 8/27 \). Show that this yields a curve with a nodal singularity. Show that the same geometric procedure for finding \( P_1 + P_2 \) on \( E \) makes the smooth points of the curve \( y^2 = 4x^3 - (4/3)x - (8/27) \) into an abelian group isomorphic to the multiplicative group \( \mathbb{C}^* \). Show that this is also isomorphic to the infinite cylinder \( \mathbb{C}/\mathbb{Z} \) and interpret this in terms of what happens to the lattice and a fundamental parallelogram.

2. (i) Let \( K \) be a field of characteristic two. Let \( E \) be the curve \( y^2 + xy = x^3 + ax^2 + b \), where \( a, b \in K, b \neq 0 \). Show that \( E \) is an elliptic curve. If \( P \) is the point \( (u, v) \) on \( E \), find a formula for \( 2P \).

(ii) For \( K \) and \( E \) as in (i), find \( E[2], K(E[2]), E[4], \) and \( K(E[4]) \). What is \([K(E[4]) : K]\) and is it always a separable extension?