The Role of Walls in Chaotic Mixing

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University of Adelaide, 22 August 2008

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Stirring and Mixing of Viscous Fluids

- Viscous flows $\Rightarrow$ no turbulence! (laminar)
- Open and closed systems
- Active (rods) and passive

Understand the mechanisms involved.
Characterise and optimise the efficiency of mixing.
Stirring and Mixing: What’s the Difference?

- **Stirring** is the mechanical motion of the fluid *(cause)*;
- **Mixing** is the homogenisation of a substance *(effect, or goal)*;
- Two extreme limits: *Turbulent* and *laminar* mixing, both relevant in applications;
- Even if turbulence is feasible, still care about energetic cost;
- For very viscous flows, use simple time-dependent flows to create *chaotic* mixing.
- Here we look at the impact of the vessel *walls* on mixing rates.
The Figure-Eight Stirring Protocol

- Circular container of viscous fluid (sugar syrup);
- A rod is moved slowly in a ‘figure-eight’ pattern;
- Gradients are created by stretching and folding, the signature of chaos.

The Mixing Pattern

- Kidney-shaped mixed region extends to wall;
- Two parabolic points on the wall, one associated with injection of material;
- Asymptotically self-similar, so expect an exponential decay of the concentration (‘strange eigenmode’ regime).

(Pierrehumbert, 1994; Rothstein et al., 1999; Voth et al., 2003)
Mixing is Slower Than Expected

Concentration field in a well-mixed central region

\[ \text{Variance} = \int |\theta|^2 dV \]

\[ \Rightarrow \text{Algebraic decay of variance} \neq \text{Exponential} \]

The ‘stretching and folding’ action induced by the rod is an exponentially rapid process (chaos!), so why aren’t we seeing exponential decay?
A Second Scenario

How do we mimic a slip boundary condition?

“Epitrochoid” protocol

Central chaotic region + regular region near the walls.
Recover Exponential Decay

\[ \sigma^2(C) \]

\( t \)
Rotating the Wall

Fixed wall: parabolic separation point (algebraic)

Moving wall: hyperbolic fixed point (exponential)
Conclusions

• If the chaotic region extends to the walls, then the decay of concentration is algebraic (typically \((\log t)/t^{-2}\) for variance).
• The no-slip boundary condition at the walls is to blame.
• Would recover a strange eigenmode for very long times, once the mixing pattern is within a Batchelor length from the edge (not very useful in practice!).
• We can shield the mixing region from the walls by wrapping it in a regular island — rotate the wall!
• We then recover exponential decay.
• How to control this in practice? Is it really advantageous? Is scraping the walls better?
• See [Gouillart et al., PRL 99, 114501 (2007); PRE (2008)]
• Thanks to Matt for use of his code!


