Do not hand in this exam.
Put all your answers on the Yellow exam.

No notes, no books, no calculators, no cell phones, no pagers, no electronic devices of any kind.

1. Let θ be the following sentence of the predicate logic:

$$\forall x\ \exists y\ (R(x,y) \wedge S(y))$$

which of the following is logically equivalent to the negation of θ, i.e.,

$$\neg\theta \equiv ?$$

(a) $\forall x\ \exists y\ (\neg R(x,y) \wedge \neg S(y))$

(b) $\exists x\ \forall y\ (R(x,y) \rightarrow S(y))$

(c) $\exists x\ \forall y\ (\neg R(x,y) \vee \neg S(y))$

(d) $\forall x\ \forall y\ \neg(R(x,y) \vee S(y))$

(e) None of above.

2. For integers n and m greater than one there exists a multiplicative inverse of m mod n, i.e., an integer k such that

$$mk \equiv 1 \bmod n$$

if and only if

(a) m and n are relatively prime

(b) m does not divide n

(c) mn is not a prime number

(d) m and n are divisible by the same prime numbers

(e) None of above.

3. Let a and b be real numbers with $0 < a < b$ use the floor and/or ceiling functions to express the number of integers n that satisfy the inequality $a < n \leq b$:

(a) $\lfloor b \rfloor - \lfloor a \rfloor$

(b) $\lceil b \rceil - \lfloor a \rfloor + 1$

(c) $\lceil b \rceil - \lfloor a \rfloor$

(d) $\lfloor b - a \rfloor$

(e) None of above.

4. Find all integers a such that

$$a \bmod 3 = a \bmod 5.$$

(a) a is either a multiple of 3 or a multiple of 5.

(b) a is 8 mod 15.

(c) a is not 3 mod 5

(d) a is $0, 1,$ or $2 \pmod{15}$.

(e) None of the above.

5. Find the greatest common divisor of $2^3\, 3^6\, 11^3$ and $2^2\, 3^7\, 5^4\, 11^5\, 13$.

(a) $2^2\, 3^6\, 11^3$

(b) $2^3\, 3^7\, 11^5$

(c) $2^2\, 3^6\, 5^4\, 11^3\, 13$

(d) $2^3\, 3^7\, 5^4\, 11^5\, 13$

(e) None of above.

6. Find the least common multiple of $2^3\,3^6\,11^3$ and $2^2\,3^7\,5^4\,11^5\,13$.

(a) $2^2\,3^6\,11^3$

(b) $2^3\,3^7\,11^5$

(c) $2^2\,3^6\,5^4\,11^3\,13$

(d) $2^3\,3^7\,5^4\,11^5\,13$

(e) None of above.

7. Suppose

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}.$$

Find A^n.

(a)

$$\begin{bmatrix} n & 0 \\ n & n \end{bmatrix}.$$

(b)

$$\begin{bmatrix} 1 & n-1 \\ n & 1 \end{bmatrix}.$$

(c)

$$\begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix}.$$

(d)

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}.$$

(e) None of above.

8. Given that $f(n)$ is defined recursively by $f(0) = 0$ and $f(n+1) = (f(n)+1)^2$ for $n = 0, 1, 2, \ldots$.
Find $f(3)$:

(a) 4

(b) 5

(c) 25

(d) 26^2

(e) None of above

9. How many ways are there to arrange the letters: g, r, e, a, t so that a is immediately followed by t?

(a) 24

(b) 60

(c) 72

(d) 96

(e) 120

(f) None of above

10. How many ways are there to arrange the letters: g, r, e, a, t so that a occurs somewhere to the left of where t occurs?

(a) 24

(b) 60

(c) 72

(d) 96

(e) 120

(f) None of above

11. How many ways are there to arrange the letters: g, r, e, a, t so that a and t do not occur next to each other (in either order)?

(a) 24

(b) 60

(c) 72

(d) 96

(e) 120

(f) None of above

12. What is the probability that when a fair coin is flipped five times in a row it comes up Heads every time?

(a) 2^5

(b) $(\frac{1}{2})^5$

(c) $(\frac{1}{5})^2$

(d) $\frac{5}{2}$

(e) None of above.

13. Suppose a fair die is rolled 180 times. Define the random variable X to be the number of times a 6 comes up. What is the expected value $\mu = E(X)$?

(a) 5

(b) 25

(c) 30

(d) 60

(e) 90

(f) None of above.

14. (same X) Suppose a fair die is rolled 180 times. Define the random variable X to be the number of times a 6 comes up. What is the variance $\nu = Var(X)$?

(a) 5

(b) 25

(c) 30

(d) 60

(e) 90

(f) None of above.

15. Suppose that A is a set with n-elements. How many binary relations are there on A?

(a) n^2

(b) 2^{n^2}

(c) 2^{n^2-n}

(d) $2^{n(n+1)/2}$

(e) $3^{n(n-1)/2} \cdot 2^n$

(f) None of above.

16. Suppose that A is a set with n-elements. How many reflexive binary relations are there on A?

(a) n^2

(b) 2^{n^2}

(c) 2^{n^2-n}

(d) $2^{n(n+1)/2}$

(e) None of above.

17. (Choose all that apply.) A binary relation R on a set A is an equivalence relation iff it is ??

(a) reflexive

(b) antisymmetric

(c) symmetric

(d) irreflexive

(e) transitive

18. (Choose all that apply.) A binary relation R on a set A is a partial order iff it is ??

(a) reflexive

(b) antisymmetric

(c) symmetric

(d) irreflexive

(e) transitive

19. Let A be the set of all bit-strings of length 16. For $x, y \in A$ define $x \equiv y$ iff x and y have the same number of 1's. How many equivalences classes are there?

(a) 16

(b) 17

(c) 2^{16}

(d) $C(16, 6)$

(e) None of above.

20. (Same $\equiv$ as above.) Let A be the set of all bit-strings of length 16. For $x, y \in A$ define $x \equiv y$ iff x and y have the same number of 1's. How many elements are there in the equivalence class which contains 0001111000011000?

(a) 6

(b) 16

(c) 17

(d) $C(16, 6)$

(e) 6!

(f) $P(16, 6)$

(g) None of above.

Put your answers to the following two problems on the Yellow exam.

Problem A.

Draw the Hasse diagram for the partially ordered set:
$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ and $R \subseteq A \times A$ defined by xRy iff x divides y.

Problem B.

Find the closed form solution of the recurrence relation:

$$a_0 = 1$$

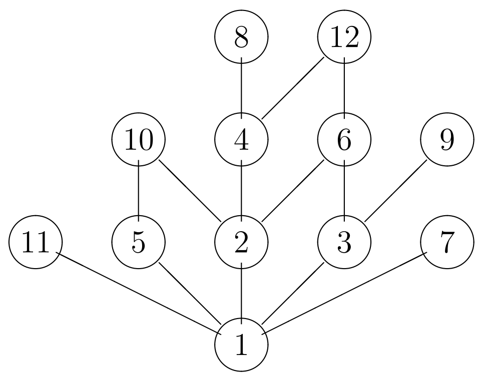
$$a_1 = 5$$

$$a_{n+1} = 2a_n - a_{n-1} + 2^n$$

Answers

1-20
c a a d a d c
c a b c b c b
b c ace abe b d

A.



B. $a_n = 2n - 1 + 2^{n+1}$