Open Eulerian trails and Hamiltonian paths and cycles

Lecture 33

(Brualdi Ch. 11.2, 11.3)

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Friday, November 20th

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475 Exam II, Covers Ch 6, 7, 8(.1-.3, ex. 1-30), Monday, November 23

closed Eulerian trail in $G$

$G$ is connected

$\forall v_i \in V,
\deg(v_i) = \text{even}$
**Theorem:** Let $G$ be a connected graph. Then $G$ has an open Eulerian trail if and only if there are exactly two vertices $u$ and $v$ of odd degree. Every open Eulerian trail in $G$ joins $u$ and $v$.

**Proof:**

$\Rightarrow$

- $\deg(a) = \text{odd}$
- $\deg(b) = \text{odd}$
- $\deg(v) = \text{even}$

$G_1 = (V \cup \{n\}, E \cup \{(u, n), (v, n)\})$

$G_1$ is connected, all vertices in $G_1$ have even degree.

$\Rightarrow G_1$ has closed Eulerian trail.

**Question:**

- $\deg(u) = \text{odd}$
- $\deg(v) = \text{odd}$

$G_2 = (V \cup \{n\}, E \cup \{(u, n), (v, n)\})$

$G_2$ is connected, all vertices in $G_2$ have even degree.

$\Rightarrow$ $G_2$ has closed Eulerian trail.
and now they build one more bridge:

Exercise
Example:

Theorem: Let $G$ be a connected graph and suppose that the number of vertices of $G$ with odd degree is $m > 0$. Then the edges of $G$ can be partitioned into $\frac{m}{2}$ open trails. It is impossible to partition the edges of $G$ into fewer than $\frac{m}{2}$ open trails.

Theorem: Let $G$ be a connected general graph having $K$ edges. Then there is a closed walk in $G$ of length $2K$ in which the number of times an edge is used equals twice its multiplicity.