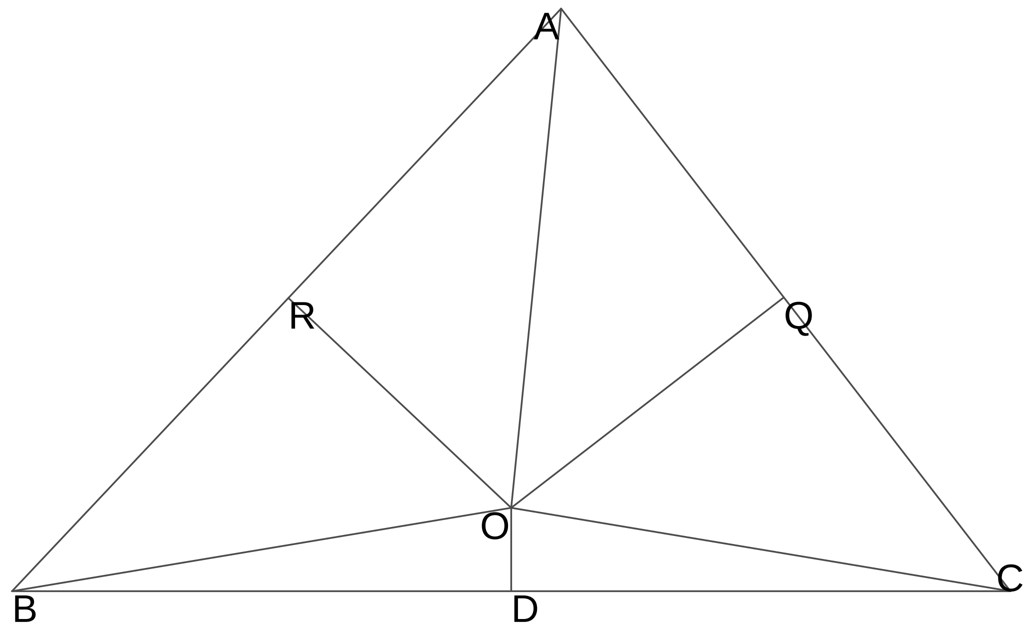
Every Triangle is Isosceles!?

Let ABC be a triangle; we will prove that $AB = AC$. Let O be the point where the perpendicular bisector of BC and the angle bisector at A intersect, D be the midpoint of BC, and R and Q be the feet of the perpendiculars from O to AB and AC respectively (see figure).



The right triangles ODB and ODC are congruent since $OD = OD$ and $DB = DC$. Hence $OB = OC$. Also the right triangles AOR and AOQ are congruent since $\angle RAO = \angle QAO$ (AO is the angle bisector) and $\angle AOR = \angle AOQ$ (the angles of a triangle sum to 180 degrees) and AO is a common side. Hence $OR = OQ$. The right triangles BOR and COQ are congruent since we have proved $OB = OC$ and $OR = OQ$. Hence $RB = QC$. Now $AR = AQ$ (as AOR and AOQ are congruent) and $RB = QC$ (as BOR and COQ are congruent) so $AB = AR + RB = AQ + QC = AC$ as claimed.