Homework 2 solutions

a 234 TA's production

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Chapter 1, section 12

10.- For this answer you will have to look at figure 11 at the book. It is in page 17. As a general idea of the question, it described us a cube of length 1 and sides over the axis $x$, $y$ and $z$.

(c) Draw the plane through $ACH$. Find a normal to the plane $ACH$

For the draw, we must do a triangle through $A$, $C$ and $H$ in the picture. Now, to find the normal vector:

Following the description at the beginning of the exercise we know the co-ordinates of the following points: $A = (0, 0, 0)$, $B = (1, 0, 0)$, $D = (0, 1, 0)$ and $E = (0, 0, 1)$.

Also, using the fact that this is a cube and that $H$ is over $D$ and $C$ is the last corner of $ABCD$ we have that $H = (0, 1, 1)$ and $C = (1, 1, 0)$.

Finally, to find the normal to the plane we need to find two vectors in the plane and compute the dot product. We have that:

$\vec{AC} = C - A = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

$\vec{AH} = H - A = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$.

So

$\vec{AC} \times \vec{AD} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \cdot 1 - 0 \cdot 0 \\ -0 \cdot 1 - 1 \cdot 1 \\ 0 \cdot 1 - 1 \cdot 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$

is normal to the plane $ACH$.

(d) Find the angle between the planes $BDE$ and $ACH$.

As explain in the exercise, to compute the angle between two planes it is enough to compute the angle between its normal vectors. We already have the normal vector of $ACH$ (the position vector of $(1, -1, 1)$). We need to compute the normal vector to $BDE$. 

1
According to the exercise $B = (1, 0, 0)$, $D = (0, 1, 0)$ and $E = (0, 0, 1)$. We will do the same as in (c): find two vectors in the plane and cross product them.

\[
\vec{BD} = D - B = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}.
\]

\[
\vec{BE} = E - B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}.
\]

\[
\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \cdot 1 - 0 \cdot 0 \\ -((-1) \cdot 1 - 0 \cdot (-1)) \\ (-1) \cdot 0 - 1 \cdot (-1) \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}.
\]

Finally, since $\vec{a} \cdot \vec{b} = \|\vec{a}\| \|\vec{b}\| \cos(\theta)$, we have that $\cos(\theta) = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \|\vec{b}\|}$. So, for use, we have that

\[
\cos(\theta) = \frac{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}}{\sqrt{1^2 + 1^2 + 1^2} \sqrt{1^2 + (-1)^2 + 1^2}} = \frac{1 - 1 + 1}{\sqrt{3} \sqrt{3}} = \frac{1}{3}
\]

This means that the angle between the planes is $\arccos\left(\frac{1}{3}\right)$.

14. - True or False

(b) If $\vec{a} \perp \vec{b}$ and also $\vec{a} \perp \vec{c}$ then $\vec{a} \perp \vec{b} + \vec{c}$?

This is true, in order to proof it, notice that:
- $\vec{a} \perp \vec{b}$ means $\vec{a} \cdot \vec{b} = 0$.
- $\vec{a} \perp \vec{c}$ means $\vec{a} \cdot \vec{c} = 0$.

With this we can compute:

\[
\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} = 0 + 0 = 0.
\]

This means that $\vec{a} \perp (\vec{b} + \vec{c})$.

(d) If $\vec{a} \perp (\vec{b} + \vec{c})$ and also $\vec{a} \perp (\vec{b} - \vec{c})$ then $\vec{a} \perp \vec{b}$?

This is also true. We can do this in two different ways (or a combination of both):

Way 1 As before we have that
- $\vec{a} \perp (\vec{b} + \vec{c})$ means $0 = \vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$.
- $\vec{a} \perp (\vec{b} - \vec{c})$ means $0 = \vec{a} \cdot (\vec{b} - \vec{c}) = \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c}$.

Now, we can add the two expressions, then:
Since $2(\vec{a} \cdot \vec{b}) = 0$ then $\vec{a} \cdot \vec{b} = 0$ so $\vec{a} \perp \vec{b}$.

Way 2 We can use (b), since $\vec{a} \perp (\vec{b} + \vec{c})$ and also $\vec{a} \perp (\vec{b} - \vec{c})$ then $\vec{a} \perp [(\vec{b} + \vec{c}) + (\vec{b} - \vec{c})]$. We can compute this last expression and we get

$$(\vec{b} + \vec{c}) + (\vec{b} - \vec{c}) = 2\vec{b}.$$ 

So we have that, $\vec{a} \perp 2\vec{b}$. Finally, since $\vec{b}$ is parallel to $2\vec{b}$ this means that $\vec{a} \perp \vec{b}$.

15.- Simplify the following expressions:

(b) $(\vec{a} + \vec{b} + \vec{c}) \times (\vec{a} + \vec{b} + \vec{c}) = 0$, since $\vec{x} \times \vec{x} = 0$ always.

(c) 

$$(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \vec{a} \times (\vec{a} - \vec{b}) + \vec{b} \times (\vec{a} - \vec{b}) = \vec{a} \times \vec{a} - \vec{a} \times \vec{b} + \vec{b} \times \vec{a} - \vec{b} \times \vec{b} = 0 + \vec{b} \times \vec{a} + \vec{b} \times \vec{a} - 0 = 2(\vec{b} \times \vec{a}).$$

(d) $(\vec{a} + \vec{b} - \vec{c}) \times (\vec{a} - \vec{b} + \vec{c})$. We will try to use (c).

$$(\vec{a} + \vec{b} - \vec{c}) \times (\vec{a} - \vec{b} + \vec{c}) = (\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) + \vec{c} \times ((\vec{a} - \vec{b}) + \vec{c})$$

$$= (\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) + (\vec{a} + \vec{b}) \times \vec{c} - \vec{c} \times (\vec{a} - \vec{b}) - \vec{c} \times \vec{c} = 2(\vec{b} \times \vec{a}) + \vec{a} \times \vec{c} + \vec{b} \times \vec{c} - \vec{c} \times \vec{a} + \vec{c} \times \vec{b} - 0 = 2(\vec{b} \times \vec{a}) + \vec{a} \times \vec{c} + \vec{b} \times \vec{c} + \vec{a} \times \vec{c} - \vec{b} \times \vec{c} = 2(\vec{b} \times \vec{a}) + 2(\vec{a} \times \vec{c}).$$

16.- This problem is about “cross division”, i.e can you solve $\vec{a} \times \vec{b} = \vec{c}$ for $\vec{b}$ if you know $\vec{a}$ and $\vec{c}$?

(a) Let $\vec{a} = \vec{e}_1 - \vec{e}_3$ and $\vec{c} = \vec{e}_1 + 3\vec{e}_2 + 2\vec{e}_3$. Find $\vec{b}$ for which $\vec{a} \times \vec{b} = \vec{c}$ if there is such a thing.

We know that, if $\vec{a} \times \vec{b} = \vec{c}$ then $\vec{a} \cdot \vec{c} = 0$ (since $\vec{a} \perp (\vec{a} \times \vec{b})$). Notice that in this case

$$\vec{a} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}; \vec{c} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}; \vec{a} \cdot \vec{c} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 3 & 3 \\ -1 & 2 & 2 \end{pmatrix} = 1 + 0 - 2 = -1.$$

Therefore, there could not exist such a $\vec{b}$.

(b) Let $\vec{a} = 2\vec{e}_1 - \vec{e}_3$ and $\vec{c} = \vec{e}_1 + 3\vec{e}_2 + 2\vec{e}_3$. Find $\vec{b}$ for which $\vec{a} \times \vec{b} = \vec{c}$ if there is such a thing.

In this case we have that
\[
\vec{a} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}; \vec{c} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}; \vec{a} \cdot \vec{c} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = 2 + 0 - 2 = 0.
\]

So, in this case, we have hope to find such a \( \vec{b} \). We know that such a \( \vec{b} \) should be perpendicular to \( \vec{c} \). We have two ways to find it:

Way 1
Since we are looking for something perpendicular to \( \vec{c} \) we should find something in the plane whose normal vector is \( \vec{c} \), in other words, the plane defined by \( x + 3y + 2z = 0 \).

We want somethin different to \( \vec{a} \) so we can ask for \( x = 0 \). After that we have that \( 3y + 2z = 0 \) if we pick \( y = 2 \) then \( 6 + 2z = 0 \) and \( z = -3 \). So the vector

\[
\begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix}
\]

is a candidate.

Nevertheless, notice that

\[
\vec{a} \times \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 0 \cdot (-3) - (-1) \cdot 2 \\ -2 \cdot (-3) - (-1) \cdot 0 \\ 2 \cdot 2 - 0 \cdot 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \\ 4 \end{pmatrix}.
\]

That is not what we want, but notice that

\[
\begin{pmatrix} 2 \\ 6 \\ 4 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = 2\vec{c}.
\]

Using the properties of cross product we know that \( \vec{b} = \frac{1}{2} \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix} \) is an answer, since

\[
\vec{a} \times \left( \frac{1}{2} \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix} \right) = \frac{1}{2} \left( \vec{a} \times \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix} \right) = \frac{1}{2} \begin{pmatrix} 2 \\ 6 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = \vec{c}.
\]

Way 2
Since we want something perpendicular to \( \vec{c} \) we may, as well, have it perpendicular to \( \vec{a} \) so we can compute

\[
\vec{a} \times \vec{c} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \cdot 2 - (-1) \cdot 3 \\ -2 \cdot 2 - (-1) \cdot 1 \\ 2 \cdot 3 - 0 \cdot 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -5 \\ 6 \end{pmatrix}.
\]

This vector is a candidate, nevertheless

\[
\vec{a} \times \begin{pmatrix} 3 \\ -5 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \times \begin{pmatrix} 3 \\ -5 \\ 6 \end{pmatrix} = \begin{pmatrix} 0 \cdot 6 - (-1) \cdot (-5) \\ -2 \cdot 6 - (-1) \cdot 3 \\ 2 \cdot (-5) - 0 \cdot 3 \end{pmatrix} = \begin{pmatrix} -5 \\ -15 \\ -10 \end{pmatrix}.
\]
That is not what we want, but notice that
\[
\begin{pmatrix}
-5 \\
-15 \\
-10 \\
\end{pmatrix}
= -5 \begin{pmatrix}
1 \\
3 \\
2 \\
\end{pmatrix}
= -5 \vec{c}.
\]

Using the properties of cross product we know that \( \vec{b} = \frac{1}{-5} \begin{pmatrix}
3 \\
-5 \\
6 \\
\end{pmatrix} \) is an answer. In order to check it we can repeat the end of way 1.

NOTE: as we can notice from answer in way 1 and way 2 \( \vec{b} \) is not necessarily unique.

Chapter 2, section 17

2.- What sign does \( \omega \) have in figure 7? How would the figure change if we change the sign of \( \omega \)? Does the force \( \vec{F} \) of the object change if we change the sign of \( \omega \)?

For this exercise you need to see figure 7 in page 27.

- In the figure, \( \omega \) is positive (going counterclockwise).

- It will change the same as we put a mirror over the \( x \)-axis: The vector would be in the third quadrant, the force would be pointing to the center, the vector would be going up-left and the movement would be going clockwise.

- This is tricky, since \( \vec{F} = -\omega^2 \vec{x} \) if \( \omega \) changes sign then \( \omega^2 \) would be the same, nevertheless, \( \vec{x} \) would change (because things are going clockwise). The force would be pointing to the center but it would be pointing up-right instead of down-right.

3.- This exercise has long presentation, please follow it at page 32 of the book

(a) Assuming the line \( \ell \) passes through the origin show from the drawing that the velocity vector of the point \( P \) is \( \vec{v} \) given by \( \vec{\omega} \times \vec{x} \).

In the problem they suggest us to show it by explaining why they have the same direction and then explaining why they have the same length.

Looking at the picture in page 32, we can see a triangle form by the line \( \ell \) the radius label \( r \) (that we will call \( \vec{r} \) and \( \vec{x} \). Which means that those three vectors are in the same plane. We will show that \( \vec{v} \) is perpendicular to that plane, then it would be parallel to \( \vec{\omega} \times \vec{x} \).

So, we know that \( \vec{v} \) is in the same plane as the circle, so it is perpendicular to \( \ell \). Also, we know that the tangent of the circular motion is perpendicular to the radius, so it is perpendicular to \( \vec{r} \). Therefore, \( \vec{v} \) is perpendicular to the plane where \( \vec{\omega} \) and \( \vec{x} \) are.

To see that it is not just parallel but in the same direction, use the picture and the right hand rule for \( \vec{\omega} \times \vec{x} \) (they have the same direction).
Now, for the length, according to the exercise the length of $\vec{v}$ is $r\omega$. We know that $\|\vec{\omega} \times \vec{x}\| = \|\vec{\omega}\|\|\vec{x}\| \sin(\theta)$. Notice that, in the picture, there is a triangle involving $\vec{x}$ and $\vec{r}$, there we can see that $\sin(\theta) = \frac{\|\vec{r}\|}{\|\vec{x}\|}$. Therefore

$$
\|\vec{\omega} \times \vec{x}\| = \|\vec{\omega}\|\|\vec{x}\| \sin(\theta) = \omega \|\vec{x}\| r = \|\vec{v}\|.
$$

(b) Show that the acceleration vector is given by $\vec{a} = \vec{\omega} \times (\vec{\omega} \times \vec{\omega})$.

We will use product rule and the fact that $\vec{\omega}$ is constant and $\vec{v} = \vec{\omega} \times \vec{x}$.

$$
\vec{a} = \frac{d}{dt} \vec{v} = \frac{d}{dt} (\vec{\omega} \times \vec{x}) = (\frac{d}{dt} \vec{\omega}) \times \vec{x} + \vec{\omega} \times (\frac{d}{dt} \vec{x}) = 0 \times \vec{x} + \vec{\omega} \times \vec{v} = \vec{\omega} \times \vec{v} = \vec{\omega} \times (\vec{\omega} \times \vec{x}).
$$

(c) Is it right to say that $\vec{a} = (\vec{\omega} \times \vec{\omega}) \times \vec{x}$? What about $\vec{\omega} \times \vec{\omega} \times \vec{x}$?

Both are wrong. In the first one we will be saying that $\vec{a} = 0 \times \vec{x}$ which is false: we have an acceleration due to the change of direction in the velocity. For the second one, since $\times$ is not associative, the expression do not makes sense.

(d) True or false: $\vec{v} \perp \vec{r}$, $\vec{a} \perp \vec{v}$ and $\vec{a}$ is parallel to $\vec{r}$?

- $\vec{v} \perp \vec{r}$ is true. In (a) we proved that $\vec{v} = \vec{\omega} \times \vec{x}$.
- $\vec{a} \perp \vec{v}$ is true. In (b) we proved that $\vec{a} = \vec{\omega} \times (\vec{\omega} \times \vec{\omega}) = \vec{a} = \vec{\omega} \times \vec{v}$.
- $\vec{a}$ and $\vec{x}$ are not parallel (so the last one is false). Since $\vec{a} = \vec{\omega} \times \vec{v}$ we have that $\vec{a}$ is perpendicular to $\vec{\omega}$, therefore it is in the same plane as $\vec{v}$ and the circle. It is clear that $\vec{x}$ is not necessary in that plane (or parallel to it).

(e) Include the acceleration vector in the drawing

The acceleration would be over the vector $\vec{r}$ pointing to the center of the circle. Knowing that the vector is in the same plane as the circle and that it is perpendicular to $\vec{v}$ we know that it is parallel to $\vec{r}$. To know the direction we need to use the right hand rule with the picture.

4.- Consider the “twisted cubic”, i.e, the curve given by $\vec{x}(t) = t\vec{e}_1 + t^2\vec{e}_2 + t^3\vec{e}_3$.

(a) Find a parametrization for the tangent line to the curve at the point where $t=1$. Where does the line intersects the $xy$-plane?

First of all, notice that $\frac{d}{dt} \vec{x}(t) = \vec{e}_1 + 2t\vec{e}_2 + 3t^2\vec{e}_3$ and

$$
\vec{x}(1) = \begin{pmatrix} 1 \\ 1^2 \\ 1^3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix},
$$

$$
\vec{x}'(t) = \begin{pmatrix} 1 \\ 2(1) \\ 3(1^2) \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.
$$
So the parametric equation of the tangent line is
\[
\begin{pmatrix}
1 \\
1 \\
1
\end{pmatrix} + s
\begin{pmatrix}
1 \\
2 \\
3
\end{pmatrix} = 
\begin{pmatrix}
1 + s \\
1 + 2s \\
1 + 3s
\end{pmatrix}.
\]

If we want to intersect that line with the \(xy\)-plane, defined by the equation \(z = 0\). It is enough to solve for \(1 + 3s = 0\), solve for \(s\) and plug it back in.

So, working on it we have that \(s = -\frac{1}{3}\) so the intersection point is
\[
\begin{pmatrix}
1 + -\frac{1}{3} \\
1 + 2\times-\frac{1}{3} \\
1 + 3\times-\frac{1}{3}
\end{pmatrix} = 
\begin{pmatrix}
\frac{2}{3} \\
\frac{1}{3} \\
0
\end{pmatrix}.
\]

(b) For any given \(t\) find the tangent line to the curve at the point \(X(t)\) and find where this curve intersects the \(xy\)-plane

Since we already have \(\vec{x}(t)\) and \(\vec{x}'(t)\) we know that the tangent line is
\[
\begin{pmatrix}
t \\
t^2 \\
t^3
\end{pmatrix} + s
\begin{pmatrix}
1 \\
2t \\
3t^2
\end{pmatrix} = 
\begin{pmatrix}
t + s \\
t^2 + 2st \\
t^3 + 3st^2
\end{pmatrix}.
\]

If we want to intersect it with the \(xy\)-plane, we must do the same as before. This time \(t^3 + 3st^2 = 0\) if \(t = 0\) then \(0 + s(0) = 0\) so \(s\) can be whatever. If \(t \neq 0\) then \(s = -\frac{t^3}{3t^2} = -\frac{t}{3}\). We will say that in any case \(s = -\frac{t}{3}\).

Now
\[
\begin{pmatrix}
t + -\frac{t}{3} \\
t^2 + 2\times-\frac{t}{3} \\
t^3 + 3\times-\frac{t}{3}
\end{pmatrix} = 
\begin{pmatrix}
t + -\frac{t}{3} \\
t^2 + 2\times-\frac{t}{3} \\
t^3 + 3\times-\frac{t}{3}
\end{pmatrix} = 
\begin{pmatrix}
\frac{2t}{3} \\
\frac{1}{3} \\
0
\end{pmatrix}.
\]

(c) If you call that intersection point \(\vec{P}(t)\), then which curve is traced out by the point \(\vec{P}(t)\) as \(t\) varies?

From the last exercise we got that
\[
\vec{P}(t) = 
\begin{pmatrix}
\frac{2t}{3} \\
\frac{1}{3} \\
0
\end{pmatrix}.
\]

Call \(x = \frac{2t}{3}\) then \(t = \frac{3x}{2}\). If we put everything in terms of \(x\) we get that
\[ P(t) = \left( \begin{array}{c}
\frac{x}{(\frac{3x}{2})^2} \\
\frac{3x}{0}
\end{array} \right) = \left( \begin{array}{c}
\frac{x}{3x^2} \\
\frac{4}{0}
\end{array} \right). \]

This tells us that the curve looks like the graph of the function \( f(x) = \frac{3x^2}{4} \), that is a parabola.

**Extra sheet HW2**

1.-

(a) Find the parametric equation of the line through (1,2,3) and direction
\[ \vec{v} = \left( \begin{array}{c}
4 \\
-5 \\
1
\end{array} \right) \]

\[ \left( \begin{array}{c}
\frac{1}{2} \\
\frac{3}{2}
\end{array} \right) + s \left( \begin{array}{c}
4 \\
-5 \\
1
\end{array} \right) = \left( \begin{array}{c}
1 + 4s \\
2 - 5s \\
3 + s
\end{array} \right) \]

(b) Where does the line from (a) intersect the plane \( x - y + z = 4 \)?

We just need to plug the parametrization of the line in the plane, therefore solve for

\[ (1 + 4s) - (2 - 5s) + (3 + s) = 4; \quad 2 + 10s = 4; \quad s = \frac{2}{10} = \frac{1}{5}. \]

So the line intersect the plane at
\[ \left( \begin{array}{c}
\frac{1}{2} \frac{1}{5} \\
\frac{2 - 5}{16} \\
\frac{3 + 1}{5}
\end{array} \right) = \left( \begin{array}{c}
\frac{9}{5} \\
\frac{1}{5} \\
\frac{16}{5}
\end{array} \right). \]

2.- Find the equation of the plane through (1,2,3) and normal vector \( \vec{v} = \left( \begin{array}{c}
4 \\
-5 \\
1
\end{array} \right) \).

\[ \left( \begin{array}{c}
4 \\
-5 \\
1
\end{array} \right) \cdot \left( \begin{array}{c}
x \\
y \\
z
\end{array} \right) = \left( \begin{array}{c}
4 \\
-5 \\
1
\end{array} \right) \cdot \left( \begin{array}{c}
1 \\
2 \\
3
\end{array} \right), \]

\[ 4x - 5y + z = 4 - 10 + 3, \]

\[ 4x - 5y + z = 3. \]
3.- Find the distance from the origin to the plane through \((1, 2, 3)\) and normal vector \(\vec{v} = \begin{pmatrix} 4 \\ -5 \\ 1 \end{pmatrix}\)

Here we will use the following formula \(d = \frac{\vec{n} \cdot \vec{AX}}{||\vec{n}||}\) where \(\vec{n}\) is the normal vector, \(A\) is a point in the plane and \(X\) is the point whose distance we are computing.

In our case \(\vec{n} = \begin{pmatrix} 4 \\ -5 \\ 1 \end{pmatrix}\), \(A = (1, 2, 3)\), \(X = (0, 0, 0)\), \(\vec{AX} = X - A = \begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix}\) so we got

\[
d = \frac{\begin{pmatrix} 4 \\ -5 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix}}{\sqrt{16 + 25 + 1}} = \frac{-4 + 10 - 3}{\sqrt{42}} = \frac{3}{\sqrt{42}} = \frac{3\sqrt{42}}{42}.
\]

4.- Find the equation of the plane through \(P(-6, 0, -7)\) and parallel to the \(xy\)-plane.

To be parallel to the \(xy\)-plane we need to have the same normal vector. The normal vector of the \(xy\)-plane is \(\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\).

\[
\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -6 \\ 0 \\ -7 \end{pmatrix},
\]

\(z = 7\).

5.- Find the parametric equation of the line of intersection between the planes in problems 2 and 4.

To find the parametric equation of a line we need a direction and a point.

Notice that the direction of the line is perpendicular to \(\begin{pmatrix} 4 \\ -5 \\ 1 \end{pmatrix}\) (since the direction is in the plane of problem 2 [with normal vector what we write]). Also, the direction is perpendicular to \(\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\) (since the direction is in the plane of problem 4 [with normal vector what we write]).

Therefore, the direction is parallel to the cross product of the two normal vectors. In our case
\[
\begin{bmatrix}
4 & -5 \\
1 & 1
\end{bmatrix}
\times
\begin{bmatrix}
0 \\
0
\end{bmatrix}
=
\begin{bmatrix}
(\begin{array}{cc}
-5 & 1 \\
4 & -1
\end{array})
\cdot 0 \\
(-1 & 0)
\end{bmatrix}
=
\begin{bmatrix}
-5 \\
-4
\end{bmatrix}.
\]

We already have the direction, now we must find a point. For that, we must find a point that solves the system of equations:

\[
\begin{cases}
4x - 5y + z = -3 \\
z = 7
\end{cases}
\]

We can simplify it to \(4x - 5y + 7 = -3\) so \(4x - 5y = -10\) if we pick \(y = 2\) (here you can pick either \(x\) or \(y\) to be whatever) then we have that

\[
x = 0; \quad x = 0.
\]

So the point \((0, 2, 7)\) is in the intersection.

Our line can be describe as

\[
\begin{bmatrix}
0 \\
2 \\
7
\end{bmatrix}
+ s
\begin{bmatrix}
-5 \\
-4 \\
0
\end{bmatrix}.
\]

6.- Given \(\begin{bmatrix} t \\ t^2 \\ t^3 \end{bmatrix}\) make a table for \(t = -3, -2, -1, 0, 1, 2, 3\). Plot the graph of this values.

\[
t = -3 \begin{bmatrix} -3 \\ 9 \\ -27 \end{bmatrix}; \quad t = -2 \begin{bmatrix} -2 \\ 4 \\ -8 \end{bmatrix}; \quad t = -1 \begin{bmatrix} -3 \\ -1 \\ -1 \end{bmatrix}; \quad t = 0 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix};
\]

\[
t = 1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}; \quad t = 2 \begin{bmatrix} 2 \\ 4 \\ 8 \end{bmatrix}; \quad t = 3 \begin{bmatrix} 3 \\ 9 \\ 27 \end{bmatrix}.
\]

For a plot of the graph you can see the image in next page.