Math 846: Crystal Bases in Algebraic Combinatorics
Lecture 001, MWF 1:20–2:10, Van Vleck B113
Syllabus for Semester I, 2019/2020

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Prerequisites: Good understanding of linear algebra.

Course Content: A crystal base is a purely combinatorial object that is used to describe representations of Lie algebras and quantum groups. In this introductory course, we will develop the theory of crystal bases from first principles, and see how they get used in representation theory. Along the way we will encounter topics such as: root systems, Kashiwara crystals, Young tableaux and their crystals, Stembridge crystals, insertion algorithms, bicrystals and the Littlewood-Richardson rule, crystals for Stanley symmetric functions, and Gelfand-Tsetlin patterns.

The lectures will be self contained and no prior knowledge of the subject is assumed. I will follow the text more or less. This course is suitable for first year graduate students. It is recommended for anyone interested in algebraic combinatorics, representation theory, Lie theory, quantum groups, and statistical mechanical models.

Course Credits: 3. Each week there will be three 50 minute lectures.

Evaluation: There are no exams. Near the end of the semester each non-dissertator student is expected to give one lecture, on a topic either from the text or a related topic of your choice. As the time approaches I will organize the speaking schedule and suggest topics.

Course goals/Learning outcomes: Master the material presented in lecture. For this I recommend the following study strategy. After each lecture do the following: for each stated definition write out numerous examples and non examples. For each stated result, write your own proof starting from first principles and without looking at your notes. It is not important if your proof matches mine or not. Done properly this strategy is easy to carry out, since every result in the course builds naturally on what came before.