
Prerequisites: Good understanding of linear algebra.

Course Content: In this introductory course we will discuss the basic concepts associated with quantum groups.

We will begin with a concrete example: the quantum group $U_q(\mathfrak{sl}_2)$. We will define this algebra via generators and relations; we will obtain a basis; we will compute the center, and we will describe the finite dimensional modules. We will discuss how $U_q(\mathfrak{sl}_2)$ is a quantized enveloping algebra for the Lie algebra $\mathfrak{sl}_2$. We will discuss how $U_q(\mathfrak{sl}_2)$ has the structure of a Hopf algebra.

With the example of $U_q(\mathfrak{sl}_2)$ in mind, we will turn our attention to the quantum group $U_q(\mathfrak{g})$, where $\mathfrak{g}$ is a finite dimensional complex semisimple Lie algebra. We will develop the theory of $U_q(\mathfrak{g})$ from first principles. Along the way we will encounter the following topics: The quantum trace; the Yang-Baxter equation; the triangular decomposition of $U_q(\mathfrak{g})$; modules for $U_q(\mathfrak{g})$; the center of $U_q(\mathfrak{g})$; the Harish-Chandra homomorphism; the Hopf algebra structure for $U_q(\mathfrak{g})$; $R$-matrices; a bilinear form which pairs the positive and negative parts of $U_q(\mathfrak{g})$; the braid group action and PBW type basis; crystal bases.

The lectures will be self contained and no prior knowledge of the subject is assumed. I will follow the text more or less. This course should be valuable to anyone interested in Lie theory, quantum groups, algebraic combinatorics, number theory, knot invariants, and statistical mechanical models.

Course Credits: 3. Each week there will be three 50 minute lectures.

Evaluation: There are no exams. Near the end of the semester each non-dissertator student is expected to give one lecture, on a topic either from the text or a related topic of your choice. As the time approaches I will organize the speaking schedule and suggest topics.

Course goals/Learning outcomes: Master the material presented in lecture. For this I recommend the following study strategy. After each lecture do the following: for each stated definition write out numerous examples and non examples. For each stated result, write your own proof starting from first principles and without looking at your notes. It is not important if your proof matches mine or not. Done properly this strategy is easy to carry out, since every result in the course builds naturally on what came before.